

Bulk Fermions in Soft Wall Models

S. Mert Aybat¹ and José Santiago^{1,2}

¹*Institute for Theoretical Physics, ETH, CH-8093, Zürich, Switzerland*

² *CAFPE and Departamento de Física Teórica y del Cosmos,
Universidad de Granada, E-18071 Granada, Spain*

Abstract

We discuss the implementation of bulk fermions in soft wall models. The introduction of a position dependent bulk mass allows for a well defined Kaluza-Klein expansion for bulk fermions. The realization of flavor and the contribution to electroweak precision observables are shown to be very similar to the hard wall case. The bounds from electroweak precision test are however milder with gauge boson Kaluza-Klein modes as light as ~ 1.5 TeV compatible with current experimental bounds.

Models with warped extra dimensions offer a new approach to solve the hierarchy problem, explaining the stability of the electroweak scale against the ultraviolet physics simply due to the geometry of space-time in these models. They also have a very appealing flavor structure, as they naturally predict hierarchical fermion masses with an extra built-in flavor protection for light fermions. Hard wall models [1], based on a slice of AdS_5 , are compatible with current constraints and a natural realization of flavor for masses of the gauge boson Kaluza-Klein (KK) modes heavier than $m_{KK} \geq 3.5 \text{ TeV}$ [2] (see [3] for a recent review). Soft wall models, which modify infrared (IR) physics by removing the IR brane and changing the gravitational background, have been shown to be compatible with current data for much lighter gauge boson KK modes $m_{KK} \geq 0.5 \text{ TeV}$ [4] when fermions are constrained to live on the UV brane. In this talk we discuss how to incorporate bulk fermions in the picture and the impact they have on constraint on the model from electroweak precision tests (EWPT) and flavor physics. More details can be found in [5] (see also [6] for other approaches to bulk fermions in soft wall models).

The soft wall is realized on an AdS_5 background

$$ds^2 = a^2(z) [dx^2 - dz^2] , \quad (1)$$

where the warp factor is $a(z) = L_0/z$ and L_0 corresponds to the inverse curvature scale of the AdS_5 space (and gives the location of the UV brane on the extra dimension $L_0 \leq z \leq \infty$). The departure from pure AdS is given by a dilaton with profile $\Phi = (z/L_1)^2$, so that the matter action reads

$$S_{\text{matter}} = \int d^5x \sqrt{g} e^{-\Phi} \mathcal{L}_{\text{matter}} . \quad (2)$$

L_1 is the scale at which the background begins to depart from pure AdS. The expected values of these scales for a natural theory of electroweak symmetry breaking (EWSB) are $L_0^{-1} \sim M_{\text{Pl}}$ and $L_1^{-1} \sim \text{TeV}$.

Let us now consider bulk fermions in this background. The action of a bulk fermion, $\Psi(x, z)$, reads

$$S = \int d^5x a^4 e^{-\Phi} \bar{\Psi} \left[i \not{\partial} + \left(\partial_5 + 2 \frac{a'}{a} - \frac{1}{2} \Phi' \right) \gamma^5 - aM \right] \Psi = \int d^5x \bar{\psi} [i \not{\partial} + \partial_5 \gamma^5 - aM] \psi, \quad (3)$$

where in the second equation we have defined

$$\psi(x, z) \equiv a^2(z) e^{-\Phi(z)/2} \Psi(x, z). \quad (4)$$

In order to obtain a sensible KK expansion of this bulk fermion with the required features, we postulate a z -dependent bulk Dirac mass of the form

$$M(z) = \frac{c_0}{L_0} + \frac{c_1}{L_0} \frac{z^2}{L_1^2}, \quad (5)$$

where $c_{0,1}$ are dimensionless constants expected to be order one. The equations of motion derived from the fermionic action read

$$i\not\partial\psi_{L,R} + (\pm\partial_5 - aM)\psi_{L,R} = 0, \quad (6)$$

where $\psi_{L,R} \equiv \frac{1 \mp \gamma^5}{2} \psi$. A standard expansion in KK modes,

$$\psi_{L,R}(x, z) = \sum_n f_n^{L,R}(z) \psi_{L,R}^{(n)}(x), \quad (7)$$

with $i\not\partial\psi_{L,R}^{(n)}(x) = m_n\psi_{R,L}^{(n)}(x)$ gives the equations for the fermionic profiles

$$(\partial_5 \pm aM)f_n^{L,R} = \pm m_n f_n^{R,L}. \quad (8)$$

The orthonormality condition

$$\int_{L_0}^{\infty} f_n^L f_m^L = \int_{L_0}^{\infty} f_n^R f_m^R = \delta_{nm}, \quad (9)$$

then gives the action as a sum over four-dimensional Dirac KK modes and possibly massless zero modes,

$$S = \int d^4x \sum_n \bar{\psi}^{(n)} [i\not\partial - m_n] \psi^{(n)}. \quad (10)$$

The first order coupled equations for the fermionic profiles can be iterated to give two decoupled second order differential equations

$$\left[\partial_5^2 \pm (aM)' - (aM)^2 + m_n^2 \right] f_n^{L,R}(z) = 0. \quad (11)$$

Inserting the expression of the metric and the mass, we get for the LH profile,

$$\left[\partial_5^2 - \frac{c_0(c_0 + 1)}{z^2} + \frac{c_1}{L_1^2} (1 - 2c_0) + m_n^2 - \frac{c_1^2 z^2}{L_1^4} \right] f_n^L = 0, \quad (12)$$

while the RH solution is identical to the LH one with the identification $c_{0,1} \rightarrow -c_{0,1}$. The normalizable solutions of the coupled linear equations can then be written as,

$$\left. \begin{aligned} f_n^L(z) &= N_n z^{-c_0} e^{-\frac{c_1 z^2}{2L_1^2}} U\left(-\frac{L_1^2 m_n^2}{4c_1}, \frac{1}{2} - c_0, \frac{c_1 z^2}{L_1^2}\right), \\ f_n^R(z) &= N_n \frac{m_n}{2} z^{1-c_0} e^{-\frac{c_1 z^2}{2L_1^2}} U\left(1 - \frac{L_1^2 m_n^2}{4c_1}, \frac{3}{2} - c_0, \frac{c_1 z^2}{L_1^2}\right), \end{aligned} \right\} \Rightarrow \text{for } c_1 > 0, \quad (13)$$

$$\left. \begin{aligned} f_n^L(z) &= -N_n \frac{m_n}{2} z^{1+c_0} e^{\frac{c_1 z^2}{2L_1^2}} U\left(1 + \frac{L_1^2 m_n^2}{4c_1}, \frac{3}{2} + c_0, -\frac{c_1 z^2}{L_1^2}\right), \\ f_n^R(z) &= N_n z^{c_0} e^{\frac{c_1 z^2}{2L_1^2}} U\left(\frac{L_1^2 m_n^2}{4c_1}, \frac{1}{2} + c_0, -\frac{c_1 z^2}{L_1^2}\right), \end{aligned} \right\} \Rightarrow \text{for } c_1 < 0, \quad (14)$$

where $U(a, b, z)$ is the confluent hypergeometric function and the normalization constants N_n are fixed by normalizing either the LH or the RH profile.

The masses and the possible presence of zero modes is determined by the boundary conditions (bc). It is easy to see (details can be found in [5]) the qualitative and quantitative equivalence of bc

$$[\pm, \pm]_{\text{hardwall}} \Leftrightarrow [\pm, \text{sign}(c_1)]_{\text{softwall}}. \quad (15)$$

Here $[\pm, \pm]$ denote the bc at the UV and IR brane, respectively, where a $+$ ($-$) means that the RH (LH) chirality has Dirichlet bc (it vanishes) at the corresponding brane. For instance, in the hard wall we have a LH (RH) chiral zero mode for $[++]$ ($--$) bc. Similarly, in the soft wall we find the following zero modes

$$f_0^{L,R}(c_0, c_1; z) = \left[\frac{L_0^{1 \mp 2c_0}}{2} E_{\pm c_0 + \frac{1}{2}} \left(\pm c_1 \frac{L_0^2}{L_1^2} \right) \right]^{-\frac{1}{2}} z^{\mp c_0} e^{\mp \frac{c_1 z^2}{2L_1^2}}, \quad (16)$$

where $E_\nu(z) = \int_1^\infty dt e^{-zt}/t^\nu$ is the Exponential Integral E function. A LH zero mode exists if $c_1 > 0$ and the UV bc is $[+]$, whereas a RH zero mode exists if $c_1 < 0$ and the UV bc is $[-]$, just as in the hard wall. Once the right boundary conditions for the existence of a chiral zero mode are imposed, we see that c_1 controls the exponential die-off in the IR whereas c_0 controls the localization of the zero mode. The same equivalence also occurs at the level of massive modes.

This KK expansion can be now used to study the phenomenology of bulk fermions in soft wall models. The most important features from the phenomenological point of view are the masses and couplings of the bulk fermions to the Higgs and gauge bosons. The masses scale according to Regge trajectories, as governed by the soft wall

$$m_n \sim \frac{\sqrt{n}}{L_1}. \quad (17)$$

This means that the spectrum of new massive fermions is more packed, with more modes accessible at colliders and more modes giving indirect contributions to EWPT at loop level. Indeed, a detailed calculation of the two most relevant observables, the T parameter and the $Zb_L\bar{b}_L$ coupling, using the results in [7] shows that up to ~ 10 modes can contribute before the results stabilize [5]. Regarding the couplings, both gauge and Yukawa couplings share the main features that make hard wall models so appealing. Gauge couplings to gauge boson KK modes are almost universal for UV localized fermions with departures from universality

proportional to the fermion masses. This guarantees a flavor protection mechanism similar to the one that makes hard wall models compatible with flavor data and a low scale of new physics with minimal tuning or structure. Similarly, Yukawa couplings naturally predict hierarchical masses and mixing angles so again the realization of flavor is as natural in soft wall models as it is with a hard wall.

We have performed a detailed fit to all relevant electroweak precision observables, including the one loop contribution from the top sector to the T parameter and the $Zb_L\bar{b}_L$ coupling. We performed the one loop calculations for these observables treating the EWSB perturbatively. We also checked the validity of this approximation, and even by including a small number of KK modes the perturbative treatment of EWSB is a good approximation up to per mille level. The result is that minimal models with a custodial protection of these two observables [8, 9] are compatible with EWPT provided the gauge boson KK modes are heavier than

$$m_n^{\text{GB}} \gtrsim 1.5 - 3 \text{ TeV}, \quad (18)$$

depending on the details of the Higgs boson [5]. Less minimal models or different backgrounds may allow even lighter KK modes. This milder constraint and the fact that masses scale with \sqrt{n} makes the LHC prospects of discovering these models very exciting.

Acknowledgments

This work has been partially supported by SNSF under contract 200021-117873, MICINN under the RyC program and project FPA2006-05294 and Junta de Andalucía projects FQM 101, FQM 437 and FQM03048.

-
- [1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370 [hep-ph/9905221].
 - [2] M. S. Carena, E. Ponton, J. Santiago and C. E. M. Wagner, Nucl. Phys. B **759** (2006) 202 [hep-ph/0607106]; Phys. Rev. D **76** (2007) 035006 [hep-ph/0701055]. R. Contino, L. Da Rold and A. Pomarol, Phys. Rev. D **75** (2007) 055014 [hep-ph/0612048].
 - [3] H. Davoudiasl, S. Gopalakrishna, E. Ponton and J. Santiago, arXiv:0908.1968 [hep-ph].

- [4] A. Falkowski and M. Perez-Victoria, JHEP **0812** (2008) 107 [0806.1737 [hep-ph]]; B. Batell, T. Gherghetta and D. Sword, Phys. Rev. D **78** (2008) 116011 [0808.3977 [hep-ph]].
- [5] S. Mert Aybat and J. Santiago, Phys. Rev. D **80** (2009) 035005 [0905.3032 [hep-ph]].
- [6] A. Delgado and D. Diego, 0905.1095 [hep-ph]; T. Gherghetta and D. Sword, arXiv:0907.3523 [hep-ph].
- [7] C. Anastasiou, E. Furlan and J. Santiago, Phys. Rev. D **79** (2009) 075003 [0901.2117 [hep-ph]].
- [8] K. Agashe, A. Delgado, M. J. May and R. Sundrum, JHEP **0308** (2003) 050 [hep-ph/0308036].
- [9] K. Agashe, R. Contino, L. Da Rold and A. Pomarol, Phys. Lett. B **641** (2006) 62 [hep-ph/0605341].